A METATHEORETICAL BASIS FOR INTERPRETATIONS OF PROBLEM-SOLVING BEHAVIOR

by

Steven J. Bartlett

Visiting Scholar in Philosophy and Psychology, Willamette University
and
Senior Research Professor, Oregon State University

Website: http://www.willamette.edu/~sbartlet

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STEVEN BARTLETT
(Saint Louis)

Introduction

Until Newell, Simon, and Shaw [1958] introduced a general theory of problem-solving in terms of concepts of information-processing and computer programming, there was no dominant viewpoint or group of techniques which could serve to unify research in problem-solving. Newell, Simon, and Shaw were particularly interested in the process of problem-solving, and suggested that computer programs could be interpreted as precise theoretical representations of human problem-solving processes. Their work has led to sophisticated studies of machine problem-solving in which machine processes have been advanced as accounts of human problem-solving behavior.

The machine model for human problem-solving has been extremely fruitful both in the practical assistance machine information-processing and problem-solving have provided for human objectives, and in the theoretical gains made in better understanding the nature of problem-solving activity. In spite of these welcome additions to our practical and theoretical knowledge, it

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has become increasingly clear that machine and human problem-solving cannot uncritically be postulated to involve equivalent or even similar processes, but that claims concerning the nature of any relation between the two kinds of problem-solving must be capable of some sort of justification.

As yet, problem-solving theorists have made little attempt to relate their results to analysis at the neurological level. Since it may be possible and useful to relate all or much of animal problem-solving behavior to corresponding neural events, it is important that research in problem-solving have available to it a sufficiently broad theoretical framework for multiple and complementary interpretations of problem-solving behavior, whether this is in terms of machine analogs, psychological observations, or neurological analyses.

Already there are signs of interest in the possibility of a general theory of problem-solving that encompasses both human and machine problem-solving. (Green [1966], Dreyfus [1972], Laszlo [1972].) Such a general theory need not begin by postulating a close relation between human and machine problem-solving, or even suggest that machine problem-solving can provide an adequate simulation model for human problem-solving processes. It is enough initially that a general theory of problem-solving offer us a theoretical framework in which consistent interpretations of the two forms of problem-solving can be developed, and commonalities and differences between them be detected and investigated.

In what follows. I will outline a general metatheory that attempts to furnish a useful basis for such an examination of interpretations of problem-solving behavior which different theories have proposed. This metatheoretical basis appears to me to hold some promise both because of its openness to hard data derived from different levels and ranges of observation, and because of its capacity for deliberate control of interpretations of problem-solving behavior, whether human or mechanical, macroscopic or microscopic. The general model suggested will turn out to be tolerant of certain disagreements about research strategies and estimations of salience while it provides a critical basis from which to assess competing interpretations of problem-solving behavior.

Theories of Problem-Solving Behavior

The Gestalt approach to problem-solving

The earliest of the contemporary theories of problem-solving behavior has derived from Gestalt psychology. Gestalt psychologists (Köhler [1925], Koffka [1935], Wertheimer [1945]) considered problem-solving to be a function of the integration of previously learned responses. They conceived of problem-solving as insightful, as the result of a relatively sudden organization of meaning, lending itself easily to generalization and to retention by the individual problem-solver. Studies made of problem-solving in this perspective have been subordinate to a holistic attitude which has stood in the way of detailed analyses of the structure of problem-solving. The Gestalt emphasis on the central role of the aha! experience in problem-solving has tended to encourage a neglect of step-by-step inferential problem-solving in which a sequence of well-defined steps (constructing a deductive proof or tracing a route through a maze, for example) seems to be more important than a sudden illumination of insight relating to the solution.

The psychometric approach

The second approach to problem-solving has been the psychometric. The central interest of this approach has been to characterize intelligence as a function of intellectual traits that contribute to general mental ability. The emphasis of psychometric theory on measurement adapts itself easily to experimental investigations and to the theoretical orientations of other approaches to problem-solving behavior. Guilford [1961], Laugherty-Gregg [1962], and Green [1964] have made intertheoretic suggestions of this sort.

The learning theory approach

The third theory of problem-solving behavior has grown out of laboratory studies of learning. Learning theorists have been keenly and adversely sensitive to speculative hypotheses and have therefore been sympathetic to behaviorism. Learning theorists have not sought to deny the existence of problem-solving processes, conceptual organizations, etc., but have believed that such concepts, which cannot at present be adequately defined in operational terms, should be avoided at this stage of research in problem-solving. There has been a tendency to hypothesize intervening processes with considerable caution, and to formulate these in terms of mediating stimulus-response processes and reinforcement hierarchies when such hypotheses do appear to have some use. (Staats [1963, 1964,1966], Skinner [1966], Goldiamond [1966].)

The information-processing approach

The fourth approach to problem-solving behavior has been advanced by recent investigations in information-processing. The idea soon developed that computer programs might serve not only as aids to human problem-solving abilities, but as theoretical representations of them. The adequacy of a given program to simulate a particular problem-solving behavior could be tested empirically by determining the degree to which the structure of the program resembled the structure of the behavior of human problem-solvers – usually made explicit through protocol analysis – and yielding the same set of solutions to specified problems. The "sufficiency" of a program so tested, then, led to the view that the program could serve as a theoretical model of corresponding dimensions of the problem-solving process. This suggestion provided a measure of conceptual control and detailed precision that had not been possible for the Gestalt, psychometric, and learning approaches to problem-solving.

Agreements and lines of division

The information-processing model for interpretations of problem-solving that was accepted early within Gestalt psychology (especially by Werthcimer [1945], Duncker [1945], De Groot [1946, 1965, 1966], Bruner-Goodnow-Austin [1956], and

Feigenbaum-Feldman [1963]) has offered the idea of algorithmic learning for the learning theorist (Goldiamond [1966], Staats [1966]), and has suggested similarities in terms of which psychometric studies might be tested (Gagné [1959, 1964, 1965a,1966] and De Groot [1966]). The information-processing attitude is inclined to oppose the holism of Gestalt psychology, to demand rigor and operationally defined concepts, and is concerned with the fine structure of problem-solving behavior. It accepts the belief in a mediating problem-solving process, does not claim such processes to be reducible to behavioral stimulus-response descriptions, and supplies an operational criterion of the adequacy of a theory through its test of "sufficiency".

Behavioral theorists have objected that information-processors are unscientific in yielding to the temptation to make guesses about a possible intervening process in problem-solving. The counter-argument has been proposed that such information-processing theories as the general problem-solver (Newell-Simon [1961]) comprise a "particular codification of the observed relationships between stimuli and responses." (Green [1966: 14]) Insofar as the theory fits the observed data, it is a theoretical representation of the behaviorist's stimulus-response relationships. And as such, it is no more, no less scientific than is the abstract encoding of data in the context of any scientific theory. The merit of the theory is determined by its ability to encode significant data adequately.

Behavioral analyses of problem-solving in terms of the language of operants, chains of association, habit family hierachies, etc., have thus disavowed any interest in the intermediate problem-solving process, opting either for "mediating responses" of the S-R form, or for a complete suspension of judgment concerning what has been regarded as the hypothetical character of the process linking stimulus and response in problem-solving behavior. Like the information-processors, behavioral analysts, too, have wished to avoid untestable hypothetical inferences. Behavioral and information - processing theorists have differed primarily in where they have seen fit to draw the line in formulating hypotheses.

Among behavioral interpreters of problem-solving it is fairly well agreed that problem-solving behavior cannot be adequately described in terms of a simple stimulus-response contingency or a chain of such contingencies. (Miller-Galanter-Pribram [1960], Miller [1962], Millenson [1964)], Green [1966].) Studies in theoretical linguistics and computer programming indicate that hierarchical organization is necessary for any adequate theory of complex verbal or problem-solving behavior. Since information-theorists are particularly concerned with the organization of units into hierarchies, this emphasis on organization brings the behavioral and information-processing approaches to problem-solving into a closer association.

From the Gestalt perspective, the stimulus-response mechanisms of the behavioral approach to problem-solving was felt to be simplistic and inadequate to account for observed behavior. At the same time, imprecisely understood concepts of cognitive organization and insight contributed to the vagueness of the Gestalt point of view. (See papers in Scheerer [1964].) Although Gestalt psychology has had little to say about the actual structural dynamics of insightful organization, the Gestalt interest in organization has been retained in both behavioral-and information-processing accounts of problem-solving.

The two competing models of problem-solving

The psychometric approach has offered useful tools for the other theorists, and the operationally reduced contribution of the Gestalt perspective has been inherited by information-processing theorists and accepted by some learning theorists. In this sense, the four theories of problem-solving behavior reflect two competing models which dominate current research in problem-solving: the animal experiment model for investigations of biological problem-solvers, useful because of the restriction to publicly observed behavior with a minimum of interference from the observer, and the information-processing model which has sought to find a precise way of representing problem-solving behavior mechanically through computer simulation.

The two models have accepted strict standards of justification of claims about problem-solving behavior. Both have reacted negatively to hypothetical inferences about such behavior. On the one hand, observations of behavior patterns and, on the other, the capacity of information-processing programs to simulate observed behavior, have been accepted in the two models as furnishing justification for their respective interpretations of problem-solving behavior. The information-processing theorist has been inclined to judge that protocol analysis, the classic method for studying human information-processing, supplies evidence for processes intervening between input and output. The behavioral theorist, however, regards human protocols as part of the behavior of the subject in relation to certain environmental contingencies, and thus as forming part of observed behavior patterns. Consequently, although protocol behavior constitutes observational evidence like any other kind of behavior, it loses its capacity for the behavioral theorist to serve as evidence for hypothetical processes inferred to exist behind that behavior. In this way, the information-processing model reflects a somewhat more complex, tolerant, flexible, and perhaps less rigorous standard of evidence.

In short, the sequence of operations that is *sufficient* from the information-processing point of view may be able to solve a given problem, but however interesting and useful this is, the sequence of operations itself is not, from the behavioral theorist's point of view, a behavioral event. Skinner [1966] has therefore concluded that problem-solving behavior which results in the solution of a problem is specific to that problem and is not relevant to the environmental conditions which determine problem-solving behavior. Problem-solving behavior is to be accounted for in the same way as behavior of any kind, i.e., as a function of environmental contingencies.

It is interesting to note that the behavioral model from which this objection comes effectively rules out the possibility of interpreting any sequence of operations – even were this sequence to be given as a behavioral event as Skinner insists – as evidence for the existence of intervening processes in the human subject. In fact, as we have seen, information-processing theorists have regarded human protocols to constitute both behavioral events and to describe sequences of operations in the human subject. Thus, the objection raised by Skinner becomes paradoxical since even the satisfaction of Skinner's requirement that sequences of operations comprise behavioral events cannot serve as evidence in his model for intervening problem-solving processes. Such an objection seems, therefore, to beg the question.

The two principal orientations toward problem-solving behavior therefore involve, on the one hand, an attempt on the part of information-processing theorists to infer the structure of an intervening cognitive process in problem-solving and, on the other, the attempt by the behavioral operant analyst to correlate the behavior of a problem-solving subject with precisely controlled environmental conditions.

The first step toward an integrated theory

The two principal theories of problem-solving behavior are complementary to the degree that problem-solving theorists of either persuasion seek to understand problem-solving behavior as a function of its sufficient conditions (information-processing) or its necessary conditions (operant analysis). The conflict between the two approaches ceases to be important from the standpoint of a general theory which seeks both to *reproduce* observed problem-solving behavior and to *describe* observed behavior in relation to environmental contingencies.

From this metatheoretical point of view, lines of controversy between Gestalt, psychometric, learning, and information-processing theorists, or in general between an exclusive concern for either the sufficient conditions or the necessary conditions of problem-solving behavior, tend to act as limitative blinders to the broad and interrelated range of questions that problem-solving behavior poses. There can be no doubt that more will be known about problem-solving behavior once we are able to give an integrated account of the situationally determined perceptual encoding processes of the individual problem-solver, and then supply a simulation model that furnishes a useful and interesting representation of the process. Here, particularly, there is no justification for a one-sided theoretical provincialism.

The nature of theoretical sufficiency

The sufficiency criterion for a general theory of problem-solving behavior is seductive: It leads, on the one hand, to a description of processes which can supply an operational program that solves certain problems. The program can therefore be interpreted as a sufficient model for that form of problem-solving. On the other hand, the sufficiency criterion appears to provide a description of the process a human problem-solver follows in solving the same problems. (Simon [1962], Green [1964], Ernst [1969], Newell-Simon [1962,1965, 1972].) It is this second interpretation of the sufficiency criterion which has encouraged confusion. (Forehand [1966: 382])

The confusion is well-known since it has arisen repeatedly in questions posed about any theory that is sufficient to account for observable data of a particular kind: Such a theory offers a rule-regulated representation of the data, but the question is often raised whether the rules expressed in the theory stand behind the observed data in the productive capacity which they have in reproducing through the theory or an algorithmic program the behavior of the system investigated. In other words, can we claim that the rules which adequately describe a process are the rules which the process follows? If a program can be developed by means of which a machine can be instructed to ride a bicycle with behavior that simulates a child's in riding the bicycle, can we infer that a child riding a bicycle is following this or a similar program? Of course we may infer this if we wish to allow for such speculative inferences. But it will be questionable that we know what we are talking about, until this inference can in some way be justified.

Hence it becomes questionable whether those processes are actually involved in a human subject which are inferred to be necessary in order to generate a solution mechanically. The problem this poses for the information-processing model is immediately evident:

Since human protocols are part of the problem-solver's behavior, any particular protocol, when conceived of as expressing a principle of causal explanation of the problem-solver's behavior, has only uncertain connection with the behavior. In short, a model for behavior is not automatically involved in the production of the behavior.

It is very likely misleading uncritically to assume that problem-solving behavior results from the subject's following that set of rules which enables a machine to simulate that behavior. It is unclear whether the behavior of a human problem-solving subject is to be interpreted as if the subject were following a set of instructions. Skinner is critical of a related issue: "One ... mistake is to suppose that behavior is always under the control of prior stimuli." (Skinner [1966: 242]) He adds that "[c]ontingencies of reinforcement shape ... behavior as they shape catching a ball – that is, without being formulated in rules" [1966: 243] – and apparently, one might be tempted to add, without being the result of following a set of rules. Skinner's outfielder no more follows a set of rules about trajectories in catching a ball than he is likely to have formulated them and quickly run over the rules in his mind while positioning himself for a flyball.

Probably the misleading character of the sufficiency criterion has been responsible for the conflict between operant analysts and information-processing theorists. Information-processors have been misled by interpreting their own analyses in the field of mechanical problem-solving as if they corresponded to productive intervening processes in human subjects. Newell and Simon consider the theory they advance as one which "posits a set of processes or mechanisms that produce the behavior of the thinking human." (Newell-Simon [1972: 9]) Since no justification is offered for so central an hypothesis, operant analysts have objected rather violently, and I believe quite correctly, to it.

However, in developing their own objections to the information-processing theorist's interpretation of problem-solving, operant analysts have, as we have seen with Skinner, constructed a counter-argument which begs the question. The issue has little if anything to do with whether or not there is behavioral evidence of sequences of operations *which can be used to describe the problem-solving process.* Protocol analysis identifies such sequences of operations by means of the behavior of problem-solving subjects, and information-processors have utilized these behavioral reports to develop effective descriptions of the problem-solving process which are capable of reproducing the behavior and of passing the simulation test of sufficiency. The question does not lie here.

The question is rather whether there is behavioral evidence *for the existence of intervening problem-solving processes in the human subject.* This question has been interpreted in the same way by both operant analysts and information-processors. This interpretation, thanks to the misleading character of the sufficiency criterion, has it that the instructions mechanically necessary to simulate human problem-solving behavior correspond to a similar set of operations in the human subject. The operant analysts have opposed this as unscientific, but have offered a sadly insufficient argument against it. In the meantime, the information-processing theorist remains unaware of the real nature of his disagreement with the behaviorist.

Their disagreement is perhaps most easily seen through an analysis of the relationship between rules and patterns of behavior. No sooner is the question, What is the relation between a rule and a pattern of behavior?, answered than the efforts are made clear of the information-processor to simulate human behavior. A specific pattern of behavior may be made intelligible by referring the pattern to a rule which can serve to generate the same pattern. The rule, in other words, meets the test of sufficiency. Being able to identify such a rule for generating a pattern of behavior is invaluable: it is in this capacity that machine algorithms assist human problem-solvers. Not only can such rules literally be productive, but they can serve, as the information-processing theorist claims, in a theoretical capacity to model human problem-solving behavior. Both of these claims are acceptable: A computer program is useful both practically, as an aid to human problem-solving, and theoretically, as one model among others for human problem-solving behavior. The trouble starts once the test of sufficiency suggests to the information-processor that he has gained something in addition to a valuable practical tool and an abstract model: that the operations employed in the model correspond to operations in the human problem-solver. When, from the standpoint of the operant analyst, Skinner asks, "How does a rule govern behavior?" [1966: 244], we may answer as the information-processing theorist ought: A rule does *not* govern behavior, but rather governs the model-theoretic representation of that behavior. Such a view reduces opposition between the behavioral and information-processing approaches to problem-solving by avoiding an unjustified interpretation of the criterion of sufficiency.

Metatheoretical remarks about justification

The behavioral and information–processing approaches to problem–solving behavior share strict standards of justification. If a critical assessment has been made of positions taken by the two approaches, it has been my intention to adhere to equivalent standards of justification. It may be useful to make these standards explicit here.

The sense of methodological rigor that the two major models of problem-solving behavior adhere to may be described as arising out of a fundamental need to know precisely what it is that one is talking about when one talks about problem-solving behavior and seeks to develop a general theory. It appears that both models associate knowing what one is talking about with the possibility of providing some justification for what one says. Insofar as this is the case, then in an equally general sense there are three methodological limitations which we are compelled to accept:

(i) that we will refuse to talk about claims or make claims which involve talking about what we *cannot* know – i.e., *cannot* justify;
(ii) that any claims we make in theorizing about problem-solving behavior must be such that we can justify claims to know what we are talking about; and

(iii) that our interest in theories of problem-solving behavior has both a negative/critical and a positive/constructive orientation: *negative*, in that claims about problem-solving behavior will be divided into two categories, according to the standard, "here, one can justify what one claims" and "here, no justification can be offered"; *positive*, in providing us with explicit criteria to judge when we know what we are talking about, as well as with a fund of claims which we are able to justify.

These three interrelated methodological limitations force our assent. What are the results if one seeks to deny them?

If I claim they are to be rejected, I either know what I am talking about, or I do not.

If I know what I am talking about, then I adhere to the model I seek to reject, and therefore I am inconsistent with my own claim.

If I do not know what I am talking about, no justification can be offered for my point of view, and it has no rational force to compel assent.

Consequently, to reject these methodological restrictions is either inconsistent (and in that sense *irrational*), or unjustifiable, hence arbitrary (and in that sense *stupid*).

However, it is important to acknowledge the fact that from this methodological basis concerning justification it does not follow that a given person or theory *ought* to accept such "rational" or "intelligent" constraints. The only rational justifications that can be given for the acceptance of rationality as a point of departure turn out to be circular. Rationality, in the sense in question, is incapable of legislating against adherence to an irrational or stupid position, except by begging the question and submitting *reasons*.

The insistence of information-processing and behavioral theorists upon strict compliance with requirements of justification has accordingly been associated with both the need to know what one is talking about in relation to the subject-matter of a particular approach to problem-solving behavior, and to know what one is talking about in talking about not knowing what one is talking about, and this in relation to a metatheoretical concern to make our theoretical standards of justification explicit. The metatheoretical interest is satisfied, in other words, by the realization that opposition to its self-imposed methodological constraints is self-defeating: The only opposition to a general account of justification that can be effective is rational opposition, use of justifiable claims, etc., and this can only undermine and defeat the intent of the opposition.

These few comments about justification will be of some use in the two sections that follow.

Problems, problem-solving, and the problem-solver

Problem-solving behavior can be characterized as a quadratic relation between (i) a set of initial conditions mentioned in posing a question that requests a value for an unknown, (ii) a context in terms of which such a request is understood, (iii) a problem-solver possessing certain problem-solving capabilities, and (iv) a set of standards by means of which the problem-solver can know when a solution has been obtained. It is possible to talk about each of these elements in a meaningful way only to the extent that each is implicitly assumed to be related to the others.

A problem is first of all an expression of ignorance. A problem therefore expresses a relationship between conditions or information known initially and some desired state of knowledge. The degree of misfit between initial conditions and desired result is a measure of ignorance. Secondly, a problem is a request to reduce this misfit to a minimum. It is by referring to some perhaps vaguely determined set of standards that a problem-solver is able to see that misfit has been minimized and that a solution has been realized.

A means-end analysis makes explicit these preconditions of problem-solving behavior through a comparison of desired result and initial state. Such a comparison defines a certain ignorance and indicates a misfit that is to be minimized between what is known and what is not known. In this sense it can be said that problems are an expression of the problem-solver's maladaptation in a given context. (Blackwell [1973a: 331] views scientific problems as maladaptations.) Problem-solving behavior is adaptive behavior that eliminates the misfit between the initial ignorance of the problem-solver and the value of an unknown which he wishes to obtain.

A description of the means by which maladaptation or misfit can be reduced identifies a problem-solver's abilities. The problem-solver's abilities are likely to include such abilities as memory (Katona [1940], McNemar [1965], De Groot [1966]), expectations, flexibility of hypothesis formation, and discrimination and organization skills (Guilford [1958], Taylor [1958], Getzels-Jackson [1962], Gagné [1966]), a capacity to recognize when the problem-solver's own performance meets some standard, some property that an acceptable solution will display (Gagné [1966]), and a set of context-relative values which set the problem-solver's task (Bar-Hillel [1964], Weizenbaum [1968], Dreyfus [1972]). Consequently, the problem-solver's abilities to solve a particular problem are themselves a function of the degree to which a problem-solver can interrelate initial conditions, context, and awareness of the general form of the solution. Successful problem-solving behavior is therefore an expression of the quadratic relation with which we began.

From this general point of view, it is perhaps a mistake to distinguish between external and internal conditions in problem-solving (as, e.g., in Gagné [1966]). Consider a computer program as a model of problem-solving behavior of a certain kind. The input is organized as a function of selective discrimination processes, hierarchies of operations, etc. Since the input is processed *in terms of* the structure of the program, it is misleading to distinguish between such "external conditions" as stimuli and instructions, as opposed to such "internal conditions" as recall, search, calculation, and determination of a solution. In the approach to problem-solving behavior proposed here, a problem considered as such is the expression of a set of initial conditions which directly relate to the problem-solver, characterized by a certain organization of data, performable operations, and perceptual capabilities, and a context organized in terms of the "program" of the problem-solver. That a problem is most fundamentally a question posed *for* a problem-solver is thus taken to be axiomatic: It makes no sense to talk about problems and problem-solving independently of the problem-solver, and thus independently of the information and the information-processing capabilities the problem-solver constitutes as a potential answerer of questions. Whether we refer to the behavior of living or mechanical problem-solvers is unimportant here.

Problem-solving behavior, involving both memory and sets of capabilities, frequently results in a change in competence with respect to related problems. It therefore is appropriate and natural to characterize problem-solving behavior both as *adaptive*, in that it seeks a reduction in degree of misfit between input and desired result, and as a pattern of *learning*. Gagné, speaking of human subjects, claims: "one of the fundamental criteria of problem-solving is that a kind of performance which could not be exhibited before the "problem" was solved *can* be exhibited after the "problem" is solved. In other words, the observed events in problem-solving comprise a change in human performance, and this in turn leads us to infer a change in human capability." (Gagné [1966: 130]; see also Craig [1953], Duncan [1959], and Gagné [1965].) (Of course a single successful instance of problem-solving does not, as Gagné goes on to observe, warrant the inference that there has been a change in problem-solving competence. The performance of a *class* of representative problems of a certain type, on the other hand, makes such an inference intelligible.)

Gagné accordingly describes human problem-solving as "an inferred change in human capability that results in the acquisition of a generalizable rule which is novel to the individual, which cannot have been established by direct recall, and which can manifest itself in applicability to the solution of a class of problems." (Gagné [1966: 132]) Although it is questionable whether the problem-solver normally learns a "rule" or learns to apply one (cf. Maier [1930], Katona [1940], Brown-Fraser [1963], and Gagné [1966]), it is clear that while some problem-solving may lead to increased competence and hence is learning, all problem-solving involves learning at least in the obvious sense that knowledge of a solution replaces ignorance. "[T]he process of problem-solving ... is irreversible in that after the problem is solved one cannot return to the original condition of concern over an unanswered question from which the problem-solving begins." (Blackwell [1976])

Problem-solving is sometimes thought to proceed in either of two ways which Skinner [1966: 249] identifies as "rational" and "intuitive". In rational problem-solving, the problem-solver is able to follow rules and to give "reasons" which both govern his behavior and describe relations between his behavior, the occasions for his behavior, and its characteristic consequences. Intuitive problem-solving, on the other hand, takes less time, is more likely to be wrong, and is unacceptable given our tendency to demand explicit, systematic accounts of problem-solving.

Useful though such a distinction may be to discriminate between, for example, computer programs which satisfy the sufficiency test and are therefore "rational" and those that do not, the distinction re-introduces the confusion between rule-governed behavior, which is a speculative matter, and rule-governed representations of behavior, which are subject to justification. If we are referring to problem-solving behavior, and not to certain model-theoretic properties of rules in our representations of behavior, perhaps it is well to say of "rationality" in problem-solving that it is "not located in any particular set of concepts or techniques but in the processes of problem-solving." (Blackwell [1973: 66]) If we insist on strict standards of justification, we can characterize problem-solving behavior as learning and as adaptive, but we cannot claim that such behavior must involve following rules which govern a simulation model of the behavior, even if the model is sufficient.

Problem-solving behavior has been described as expressing a pragmatical relation between a set of contingencies which make a problem specific relative to the context in which the problem is embedded, and the perceptual encoding and processing of data that constitute a functional representation of the abilities of the individual problem-solver. It is apparent that an adequate approach to problem-solving behavior requires such an integrated account. The need and usefulness of a unified theory will no doubt become increasingly evident as attempts are made to establish other transformation schemata from the language of contextually determined principles of perceptual encoding to, for example, the language of neurological events.

Problem-solving and perception

I have claimed that problem-solving behavior cannot, if we adhere to strict standards of justification, be said to entail following a set of rules, even if those rules are themselves required to effect a simulation of a certain behavior. I have argued that the claim must be based on an inference, that no justification for such an inference is available, and that hence the claim is purely speculative and should therefore be avoided. A stronger claim, however, can be made. It is this: Not only is no justification for such an inference available, it is not forthcoming. It is, I would submit, an inference for which justification is in principle impossible, and we ought not then to waste time searching for it. The argument supports the behaviorist's criticism of the information-theorist's belief to have disclosed the rules, or an equivalent set of rules, which produce the behavior of a living problem-solver.

Suppose we begin with the general question, Is it possible for the information-processing theorist to know what he is talking about when he asserts that a rule produces the pattern of behavior it can be used to reproduce? If our answer to the question is "yes", then we claim to know what we are talking about in saying that the rules necessary to generate a solution to a problem mechanically are rules actually involved in a human subject, whose behavior is simulated by the machine's. If our answer to the question is "no", then the previous answer ought not to be defensible. Accordingly, if we can show that an affirmative reply to the question is indefensible, we are justified in giving a negative answer to the question.

Here is why the information-processor's claim is unjustifiable in principle: If the affirmative reply were defensible, it would be possible to justify the claim that the rules necessary to generate a solution mechanically are rules actually involved in the human subject. What could count as justification for the claim? Whatever evidence we obtain concerning the human subject's problem-solving behavior will be acquired either through observations of his behavior (i.e., via protocol analysis, etc.) or by other means. If through observations of his behavior, we should need to justify the claim that some observed part of his total problem-solving behavior is involved as the productive basis for that behavior. To justify this claim, we should in turn be forced to make other behavioral observations or seek for justification elsewhere. In the former case, to avoid begging the question through a perpetual regress, a given observation or set of observations must be permitted to count as justification. But to allow a given behavioral observation to count as justification of the inference in question is also to beg the question, since what must be justified is the claim that this can be done. On the other hand, if justification is sought other than by recourse to observations of behavior, we should need to justify the further claim that non-behavioral considerations may be appealed to in inferring certain behaviors to express the productive basis for others. And it is just this appeal to a productive basis for problem-solving behavior that is in question. Consequently, it is in principle impossible to justify the inference to a productive role of rules in problem-solving behavior.

If this is so, in what terms ought problem-solving behavior to be described? The case for mechanical problem-solving is comparatively straight-forward. A set of instructions and operations in a computer program is successfully correlated with the machine's capacity to reproduce desired problem-solving behavior. For a biological problem-solver, it may be possible to correlate neural processes with an organism's capacity to reproduce the desired problem-solving behavior. If this can be done, a mechanical interpretation is suggested for living systems.

However, unlike automatic systems developed to date, a complete inventory of human behavior includes certain inescapable facts that are fundamental to the metatheory proposed here. The general theory suggested by me, or a theory expressed by any other living system, constitutes part of that system's behavior. When the theory is understood in a way so as to supply a framework in terms of which the behavior of the living system is rendered intelligible, we note a variety of self-reference, and it is this self-reflexive proper-

:y of behavioral theory which cannot be treated by means of the nachine model. It is precisely this fact that led to difficulties in ustifying the information-processor's claim discussed above.

We have, then, a rather peculiar problem: How to describe problem-solving behavior, including the theorist's behavioral expression of his theory of problem-solving, in a manner so that we may still claim to know what we are talking about. In what terms ought the theorist to furnish an adequate account of his own behavior?

To be more specific, theorists of whatever orientation, and problem-solving theorists in particular, are inclined to refer to solutions to problems as "discoveries", or as "creations" or "inventions". A solution to a problem is said by some to be "discovered" (Bruner [1961], Polya [1962-65], Skinner [1966: 235, 247]), while others refer to problem-solving as a creative process (Guilford [1958], Taylor [1958], Getzels-Jackson [1962], Gruber-Terrell Wertheimer [1962], Taylor [1964]). These positions are closely paralleled by the realist-logicist and the intuitionist approaches to mathematics. The views are remarkably tenacious, the first having been held by Plato and more recently by Frege and Gödel, and the second by Kant, and currently by L.E.J. Brouwer and A. Heyting. The difference of opinion, whether in research in problem-solving or in the foundations of mathematics, has been expressed in conflicting answers to the question, Is the solution of mathematical problems or of problems, generally, a matter of human discovery or of human creativity? Is the solution autonomous of human industry, in which case obtaining the solution is a matter of discovery, or is the solution dependent upon human faculties, in which case it is a creative invention?

The disclosure of a solution to a problem is thought to constitute a "discovery" if there is a sense in which the solution "was already there". Thus, Columbus is said to have "discovered" America because America was there before he arrived on its shores; a problem-solver may be said to "discover" the solution to three simultaneous equations in three unknowns because the values of the unknowns were already given implicitly by the equations, and in that sense "were already there". To call the solution to a problem a result of the "creative" faculties of the problem-solver is to make a contrary claim, that the solution "was not already there" but rather was obtained through the "inventive" resources of the problem-solver.

How problem-solving is interpreted is significant: If a theorist conceives of problem-solving on the discovery model, he is likely to attempt to formulate objective heuristic principles, techniques, and informative guidelines which can furnish problem-solvers with strict methods or liberal suggestions enabling them to reach solutions. If a theorist interprets problem-solving as essentially a creative process, on the other hand, he is likely to devote himself to a study of the psychology of invention, to attempt to gain some understanding of characteristics common to creative problem-solvers. For the theorist predisposed to accept the creative model, if problem-solving were a matter of discovery, of applying relevant existing mental rules, it would be likely to be thought *too routine* to qualify as legitimate problem-solving. (Gagné [1966: 129]) For the discovery oriented theorist, however, "creative processes" are likely to be thought *too vague* to provide a systematic basis for an understanding of problem-solving. One approach leads to a psychology of the biological subject, the other to investigations of artificial intelligence. The question here is: Can either view be justified?

The two views come into conflict in connection with the alleged status of the solution to a problem before the subject solves the problem. If there is some justifiable sense in which the solution "was already there", then support can be given for the discovery model. If justification can be provided for the view that the solution "was not already there", then in this sense it can be said that the problem-solver was creative.

Two things are immediately evident and trivial: For any problem-solver a problem expresses his ignorance of a solution. In this obvious sense, the solution "was not already there" for him before he solves the problem. Similarly, for other problem-solvers (e.g., the problem-solving theorist) a problem with a known solution, but raised for a particular problem-solver, has a solution that "was already there" for those who knew the solution beforehand. In these trivial senses, every problem is solved "creatively", and some of the solutions are "discovered".

The question is more interesting with regard to the solution of a problem not known to have been solved before. Is such a solution a "discovery" or an "invention"? If we can reply that the solution qualifies as one rather than the other, and know what we are talking about, then some justification can be provided for our claim.

But in neither case is justification possible. Whether a solution is "discovered" or "invented" is settled by reference to the status of the solution before the problem was solved. Did the solution exist, implicitly or explicitly, in any sense before the problem was solved, by our hypothesis, for the first time? Given that solving a problem provides us with knowledge of its solution, then prior to solving the problem no knowledge concerning the solution is possible. It follows, then, that prior to the solution of a problem no knowledge regarding the "existence of the solution, implicitly or otherwise" is possible. In this sense, and this is the non-trivial sense of alleged conflict between the two views in question, any attempt to talk about conditions of "discovery" as opposed to "invention" fails to be acceptable in relation to our criteria of justification. The conclusion is uncompromising: In neither case can the problem-solving theorist literally be said to know what he is talking about when he claims that problem-solving is a matter of discovery or of invention.

A group of terms and concepts neutral with respect to this speculative and fundamentally unjustifiable controversy is needed in terms of which problem-solving behavior can be characterized. Such a set of concepts will be unlikely to constrain problem-solving research either to the dimension of psychology of invention or to machine problem-solving. Since we make no assumptions about problems or about their solutions independently of their relativity to the individual problem-solver, and avoid the distinction between "internal" and "external" events in problem-solving, how can we express the relation between a solution to a problem and the individual problem-solver, given that the relation is not, as we have seen, one of "discovery" or "invention" except in the trivial sense?

It seems both natural and useful, I believe, to describe the relation in the terminology suggested by the language of perception. As the Gestaltists have proposed, the perceptual encoding processes of the individual problem-solver (although they do not speak this way) furnish him with a basis for solving problems. The same claim is now found in information-processing accounts of problem-solving behavior and pattern recognition, and is the route already taken by studies of neurological encoding processes involved in perception. At the very least, and in the most general sense, considering a problem, solving the problem, and realizing the solution is satisfactory, all reflect different ways in which the problem-solver *perceives* the problem, *sees* its solution, and *sees* that the solution can be checked.

A perceptual account of problem-solving behavior is not only theoretically unrestrictive but it appears to be confirmed by recent studies of problem-solving. Gagné [1966], for example, claims that "individual difference variables", which identify how one subject is a better problem-solver than another, are *perceptual* factors that reflect such problem-solving parameters as number of previously learned skills (Gagné says "rules"), recall, fluency in hypothesis formation, and the abilities to discriminate between salient and unnecessary information, to retain the solution model, and to generalize that model.

Speaking of De Groot's contributions to research in problem-solving, Forehand [1966: 369] observes that the "question of how expert problem-solvers differ from weak ones has been refined: the question is now posed not in terms of differences in general knowledge of the structure of the problem, but *in terms of differences in the way in which perception of a complex stimulus array is encoded and retained.*"*

Through De Groot's work it is now known that human beings do not investigate possibilities in a systematic manner, but tend rather to oscillate back and forth among what are *perceived* to be meaningful sequences, attending to variations through a process De Groot calls "progressive deepening". (Cf. De Groot [1946], [1965],

* My emphasis.

[1966].) This *ability*, which there is good reason, and none not, to term *perceptual*,constitutes the human problem-solver's present advantage over, for example, computer chess programs.

Problem-solving has also been described in perceptual terms as involving a narrowing of attention to salient possibilities. (Craig [1953], Gagné-Brown [1961], Gagné [1965b]) Protocols from subjects able to solve the famous nine dot problem or Maier's [1930] pendulum problem describe their solutions in terms of an awareness of constraints their own perception of the problem misleadingly introduced. Whether perceptual abilities sometimes narrow attention to salient possibilities or at other times broaden attention to include an awareness of misleading perceptual constraints, the case for perception in problem-solving appears to be strengthening.

In fact, were it not for its association with unsystematic approaches, a general theory of problem-solving behavior might suitably be termed "phenomenology of problem-solving behavior", only that, as Austin observed in a different context, is rather a mouthful. (Austin [1970: 182])

Summary

It has been the purpose of this paper to identify some of the characteristics of the principal models of problem-solving behavior which are useful in developing a general theory of problem-solving. An attempt has been made both to make explicit those disagreements between theorists of different persuasions which have served as obstacles to an integrated approach, and to show that these disagreements have arisen from a number of conceptual confusions: The conflict between information-processors and behavioral analysts has resulted from a common failure to understand theoretical sufficiency, and hence these theorists have been at a loss to understand one another. Two directions of research in problem-solving, mechanical, algorithmic problem-solving and psychology of invention, have been thought to be divergent, but in fact complement one another once it is clear that problem-solving involves neither discovery nor invention, but rather is a matter of perceptual encoding and processing. It has been suggested that the distinction between external and internal events has been unprofitable in research in problem-solving, and that problem-solving is most generally understood in relational terms. From this perspective, successful problem-solving behavior can be described as adaptive, learning behavior in which organization skills are effectively associated with situationally determined perceptual encoding processes of the individual mechanical or biological problem-solver.

REFERENCES

Austin [1970]. J.L. Austin: *Philosophical Papers* (Oxford: Oxford University Press, 1970 (second ed.; first ed., 1961)).

Bar-Hillel [1964]. Y. Bar-Hillel: "The Present Status of Automatic Translation of Language", in F.L. Alt, ed.: *Advances in Computers* (New York: Academic Press, 1964), vol. I.

Blackwell [1973]. Richard J. Blackwell: "Toulmin's Model of an Evolutionary Epistemology", *The Modern Schoolman*, vol. LI, no. 1, 1973, pp. 62-68.

Blackwell [1973a]. Richard J. Blackwell: The Adaptation Theory of Science", *Internatl. Philosophical Quarterly*, vol. XIII, no. 3, 1973, pp. 319-334.

Blackwell [1976]. Richard J. Blackwell: "Scientific Discovery and the Laws of Logic", Summer, 1976, *The New Scholasticism*.

Brown-Fraser [1963]. R. Brown and C. Fraser : "The Acquisition of Syntax", in C.N. Cofer and B.S. Musgrove, eds.: *Verbal Behavior and Learning* (New York: McGraw-Hill, 1963), pp. 158-196.

Bruner [1961]. J.S. Bruner: "The Act of Discovery", *Harvard Educ. Rev.*, vol. 31, 1961, pp. 21-32.

Bruner-Goodnow-Austin [1956]. J.S. Bruner, J.J. Goodnow, and G.A. Austin: *A Study of Thinking* (New York: Wiley, 1956).

Craig [1953]. R.C. Craig: *The Transfer Value of Guided Learning* (New York: Columbia University Press, 1953).

De Groot [1946]. A.D. De Groot: *Het Denken van den Schaker* (Amsterdam: N.H. Uitg. Mij. 1946).

De Groot [1965]. A.D. De Groot: *Thought and Choice in Chess* (The Hague: Mouton, 1965).

De Groot [1966]. A.D. De Groot: "Perception and Memory versus Thought: Some Old Ideas and Recent Findings", in Kleinmuntz [1966: 19-50].

Dreyfus [1972]. H.L. Dreyfus: *What Computers Can't Do: A Critique of Artificial Intelligence* (New York: Harper and Row, 1972).

Duncan [1959]. C.P. Duncan: "Recent Research on Human Problem-Solving", *Psych. Bull.*, vol. 56, 1959, pp. 397-429.

Duncker [1945]. K. Duncker: "On Problem-Solving", *Psychol. Monogr.*, 58 (5), Whole No. 270.

Ernst [1969]. G.W. Ernst: "Sufficiency Conditions for the Success of GPS", *J. of the ACM*, vol. 16, 1969, pp. 517-533.

Feigenbaum-Feldman [1963]. E.A. Feigenbaum and J. Feldman, eds.: *Computers and Thought* (New York: McGraw-Hill 1963).

Forehand [1966]. G.A. Forehand: "Constructs and Strategies for Problem-Solving Research", in Kleinmuntz [1966: 355-383].
Gagné [1959]. R.M. Gagné: "Problem-Solving and Thinking", *Ann. Rev. Psych.*, vol. 10, 1959, pp. 147-172.
Gagné [1964]. R.M. Gagné: "Problem-Solving", in A.W. Melton, ed.: *Categories of Human Learning* (New York: Academic Press 1964), pp. 293-317.
Gagné [1965]. R.M. Gagné: "The Analysis of Instructional Objectives for the Design of Instruction", in R. Glaser, ed.: *Teaching Machines and Programmed Learning*, II. *Data and Directions* (Washington, D.C.: National Education Association. 1965).
Gagné [1965a]. R.M. Gagné: *The Conditions of Learning* (New York: Holt, Rinehart and Winston, 1965).
Gagné [1966]. R.M. Gagné: "Human Problem-Solving: Internal and External Events", in Kleinmuntz [1966: 128-148].
Gagné-Brown [1961]. R.M. Gagné and L.T. Brown: "Some Factors in the Programming of Conceptual Learning", *J. Exp. Psychol.*, vol. 62, 1961, pp. 313-321.
Getzels-Jackson [1962]. J.W. Getzels and P.W. Jackson: *Creativity and Intelligence* (New York: Wiley, 1962).
Goldiamond [1966]. I. Goldiamond: "Perception, Language and Conceptualization Rules", in Kleinmuntz [1966: 183-224].
Green [1964]. B.F. Green: "Intelligence and Computer Simulation", *Transactions of the New York Academy of Science*, 1964, Ser. II, 27, pp. 55-63.
Green [1966]. B.F. Green: "Current Trends in Problem-Solving", in Kleinmuntz [1966: 3-18].
Gruber-Terrell-Wertheimer [1962]. H.E. Gruber, G. Terrell, and M. Wertheimer, eds.: *Contemporary Approaches to Creative Thinking* (New York: Atherton Press, 1962).
Guilford [1958]. J.P. Guilford: "Basic Traits in Intellectual Performance", in C.W. Taylor, ed.: *The Second University of Utah Research Conference on the Identification of Creative Scientific Talent* (Salt Lake City: University of Utah Press, 1958).
Katona [1940]. G. Katona: *Organizing and Memorizing* (New York: Columbia University Press, 1940).
Kleinmuntz [1966]. B. Kleinmuntz, ed.: *Problem Solving – Research, Method, and Theory*, (New York: Wiley, 1966).
Köhler [1925]. W. Köhler: *The Mentality of Apes* (New York: Harcourt Brace, 1925).
Koffka [1935]. K. Koffka: *Principles of Gestalt Psychology* (New York: Harcourt Brace, 1935).
Laugherty-Gregg [1962]. K.R. Laugherty and L.W. Gregg: "Simulation of Human Problem-Solving Behavior". *Psychometrika*. vol. 27, 1962, pp. 265-282.
Laszlo [1972]. E. Laszlo: *Introduction to Systems Philosophy: Towards a New Paradigm of Contemporary Thought* (New York: Gordon, 1972).
Maier [1930]. N.R.F. Maier: "Reasoning in Humans: I. On Direction", *J. Comp. Psych.*, vol. 10, 1930. pp. 115-143. "II. The Solution of a Problem and its Appearance in Consciousness". *J. Comp. Psych.*, vol. 12. 1931, pp. 181-194.
McNemar [1965]. O. McNemar: "Lost: Our Intelligence?", *Amer. Psychologist*, vol. 19, 1964. pp. 871-882.
Millenson [1964]. J.R. Millenson: "On the Relationship between Stimulus-Response Notation and Flow Diagram Notation as Representative of Algorithmic Problem-Solving" (Pittsburgh, Pa.: Carnegie Institute of Technology, Department of Psychology (CIP Paper No. 71), June 22. 1964: Ditto).
Miller [1962]. G.A. Miller: "Some Psychological Studies of Grammar", *Amer. Psychologist*. vol. 17, 1962, pp. 748-762.
Miller-Galanter-Pribram [1960]. G.A. Miller, E. Galanter, and K.H. Pribram: *Plans and the Structure of Behavior* (New York: Holt, Rinehart and Winston 1960).

Newell-Simon [1961]. A. Newell and H.A. Simon: "The Simulation of Human Thought", in W. Dennis, ed.: *Current trends in Psychological Theory* (Pittsburgh, Pa.: University of Pittsburgh Press, 1961).

Newell-Simon [1962]. A. Newell and H.A. Simon: "Computer Simulation of Human Thinking and Problem-Solving", in M. Greenberger, ed.: *Management and the Computer of the Future* (New York: Wiley, 1962), pp. 94-113.

Newell-Simon [1963]. A. Newell and H.A. Simon: "GPS, A Program that Simulates Human Thought", in E.A. Feigenbaum and J. Feldman, eds.: *Computers and Thought* (New York: McGraw-Hill, 1963).

Newell-Simon [1963a]. A. Newell and H.A. Simon: "Computers in Psychology", in Luce, Bush, and Galanter, eds.: *Handbook of Mathematical Psychology*, vol. I (New York: Wiley, 1963), pp. 361-428.

Newell-Simon [1965]. A. Newell and H.A. Simon: "Programs as Theories of Higher Mental Processes", in R.W. Stacey and B. Waxman, eds.: *Computers in Biomedical Research*, vol. II (New York: Academic Press), pp. 141-172.

Newell-Simon [1972]. A. Newell and H.A. Simon: *Human Problem-Solving* (New Jersey: Prentice-Hall, 1972).

Newell-Simon-Shaw [1958]. A. Newell, H.A. Simon, and J.C. Shaw: "Elements of a Theory of Human Problem-Solving", *Psychol. Rev.*, vol. 65, 1958, pp. 151-166.

Polya [1962-65]. G. Polya: *Mathematical Discovery: On Understanding, Learning, and Teaching Problem-Solving* (New York: John Wiley, 1962 (vol. I), 1965 (vol. II)).

Scheerer [1964]. C. Scheerer, ed.: *Cognition: Theory, Research, Promise* (New York: Harper and Row, 1964).

Simon [1962]. H.A. Simon: "An Information-Processing Theory of Intellectual Development", in W. Kessen and C. Kuhlman, eds.: *Thought in the Young Child; Monographs of the Society for Research in Child Development*, 27 (Yellow Springs, Ohio: Antioch Press, 1962), pp. 150-161.

Skinner [1966]. B.F. Skinner: "An Operant Analysis of Problem-Solving", in Kleinmuntz [1966: 225-257].

Staats [1963]. A.W. Staats and C.K. Staats: *Complex Human Behavior* (New York: Holt, Rinehart, and Winston, 1963).

Staats [1964]. A.W. Staats, ed.: *Human Learning* (New York: holt, Rinehart, and Winston, 1964).

Staats [1966]. A.W. Staats: "An Integrated-Functional Learning Approach to Complex Human Behavior", in Kleinmuntz [1966: 259-339].

Taylor [1958]. C.W. Taylor: "Some Variables Functioning in Productivity and Creativity", in C.W. Taylor, ed.: *The Second University of Utah Research Conference on the Identification of Scientific Talent* (Salt Lake City: University of Utah Press, 1958).

Taylor [1964]. C.W. Taylor, ed.: *Creativity: Progress and Potential* (New York: McGraw-Hill, 1964).

Weizenbaum [1968]. J. Weizenbaum: "Contextual Understanding of Computers", in Kolers and Eden, eds.: *Recognizing Patterns: Studies in Living and Automatic Systems* (Cambridge, Mass.: MIT Press, 1968).

Wertheimer [1945]. M. Wertheimer: *Productive Thinking* (New York: Harper and Row, 1945).